exposure time enhancement and material thickness reduction, are uniquely and completely quantified by the nondimensional parameters; χ, H, and ψ_1 in thin-wall thermal response, and χ, H, and ψ_2 in semi-infinite and finite-wall thermal response. Maximum exposure times and minimum material thickness for various incident irradiation, thermal reradiation, and thermal convection conditions can be determined directly from these nondimensional parameters and thermal response solutions discussed above. Radiative and convective cooling effects are material-dependent and not necessarily negligible for some radiative and convective conditions.

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Temperature and Heat Transfer Solutions for Aeromagnetic Dusty-Gas Flow

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Introduction

TWO-PHASE (particle-fluid) suspensions occur in many industrial processes. Understanding such processes requires the analysis of the basic equations governing multiphase flows. These equations are given by Soo[1] and Marble.[2] Rossow[3] reported equations governing the flow of an electrically conducting fluid in the presence of a magnetic field. Aeromagnetic dusty-gas flows are of interest because they have many applications in the geophysical and astronomical sciences.

In a previous paper, Chamkha[4] reported exact solutions for hydromagnetic (or more correctly aeromagnetic) flow of a particulate suspension past an infinite porous plate. In his work, Chamkha[4] did not consider the thermal energy transport equations of the suspension. The purpose of this article is to study the effect of a transverse magnetic field on the temperature profiles, and the wall heat transfer for flow of a dusty gas past an infinite porous flat plate. The fluid phase is assumed to be incompressible and electrically conducting, and the particle phase is assumed to be incompressible and electrically nonconducting. The volume fraction of suspended particles is assumed to be small. It is also assumed that there is no radiative heat transfer from one particle to another and that the particles do not interact with each other.

Governing Equations

Consider steady laminar particle-fluid flow past an infinite porous flat plate. The flow is a uniform stream parallel to the x, y plane with the plate being coincident with the plane $y = 0$. Far from the plate, both phases are in equilibrium moving with a velocity V_∞ and a temperature T_∞ in the x direction. Let uniform fluid-phase suction with velocity V_0 be imposed at the plate surface. Since the plate is infinite, the physical variables will only depend on y (the distance above the plate).

In the present problem, it is assumed following Gupta[5] that no applied voltages exit. This corresponds to the case where no energy is being added or extracted from the fluid by electrical means. The magnetic effects are confined to retarding the flow and dissipating kinetic energy into internal energy. In general, the electrical current flowing in the fluid gives rise to an induced magnetic field that distorts the applied magnetic field. This would exist if the fluid were an electrical insulator. However, since the viscous boundary layer is thin, the induced magnetic field will be neglected compared to the constant applied field acting along the y axis and moving past the plate with the freestream velocity.

The governing equations for the problem under investigation are based on the balance laws of mass, linear momentum, and energy for both phases. These can be written in dimensional form, taking into account the assumptions mentioned earlier, as

$$-\rho V_0 u' = \mu u'' + \rho_p(u_p - u)/\tau_v - \sigma B_0^2(u - V_\infty) \quad (1)$$

$$-\rho_p V_0 u_p' = -\rho_p(u_p - u)/\tau_V \quad (2)$$

$$-\rho c V_0 T' = kT'' + \mu(u')^2 + c_p\rho_p(T_p - T)/\tau_T + \rho_p(u_p - u)^2/\tau_V + \sigma B_0^2(u - V_\infty)^2 \quad (3)$$

$$-\rho_p c_p V_0 T_p' = -\rho_p c_p(T_p - T)/\tau_T \quad (4)$$

where u is the fluid-phase velocity in the x direction, u_p is the particle-phase velocity in the x direction, ρ is the fluid-phase density, ρ_p is the particle-phase density, μ is the fluid-phase dynamic viscosity, V_0 is the suction velocity, σ is the electrical conductivity, B_0 is the magnetic induction, T is the fluid-phase temperature, T_p is the particle-phase temperature, k is the fluid-phase thermal conductivity, c is the fluid-phase specific heat at constant pressure, c_p is the particle-phase specific heat, τ_v and τ_T are the velocity relaxation time and the temperature relaxation time, respectively, and a prime denotes ordinary differentiation with respect to y. The minus signs appearing on the left side of Eqs. (1–4) result from the fact that the suction velocity is acting downward in the opposite direction of the positive y direction.

The last terms appearing in Eqs. (1) and (4) are present due to the presence of the magnetic field. The former rep-

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resents the force on the fluid due to the interaction of the electric current in the fluid and the magnetic induction, and the latter represents the heat added by the electrical current produced by the motion of the fluid through the magnetic line of force.

Substituting the following equations:

$$y = \mu\eta/(\rho V_\infty), \quad u = V_\infty F(\eta), \quad u_p = V_\infty F_p(\eta)$$

$$T = T_\infty G(\eta), \quad T_p = T_\infty G_p(\eta) \tag{5}$$

into Equations (1–4) and rearranging yield

$$F'' + r_v F' + \kappa\alpha(F_p - F) + M^2(1 - F) = 0 \tag{6}$$

$$r_v F_p' + \alpha(F - F_p) = 0 \tag{7}$$

$$G'' + r_v P_r G' + P_r \kappa\gamma\varepsilon(G_p - G) + E_c P_r (F')^2$$
$$+ E_c P_r \kappa\alpha(F_p - F)^2 + M^2 P_r E_c (F - 1)^2 = 0 \tag{8}$$

$$r_v G_p' + \varepsilon(G - G_p) = 0 \tag{9}$$

where a prime denotes ordinary differentiation with respect to η and

$$r_v = V_0/V_\infty, \quad \kappa = \rho_p/\rho, \quad \alpha = \mu/(\rho\tau_v V_\infty^2)$$
$$M = \sqrt{\sigma\mu} B_0/(\rho V_\infty), \quad P_r = \mu c/k, \quad \gamma = c_p/c$$
$$\varepsilon = \mu/(\rho\tau_T V_\infty^2), \quad E_c = V_\infty^2/(cT_\infty) \tag{10}$$

are the suction parameter, the particle loading, the velocity inverse Stokes number, the Hartmann number, the fluid-phase Prandtl number, the specific heat ratio, the temperature inverse Stokes number, and the Eckert number, respectively.

Equations (6–9) will be solved subject to boundary conditions suggested by the physics of the problem. These are

$$F(0) = 0, \quad F(\infty) = 1, \quad F_p(\infty) = 1, \quad G(0) = G_0$$
$$G(\infty) = 1, \quad G_p(\infty) = 1 \tag{11}$$

where G_0 is a constant dimensionless temperature of the plate surface.

The wall heat transfer coefficient is an important physical property. This is defined in the original variables as

$$C_q = -kT'(0) \tag{12}$$

The dimensionless form of Eq. (12) in the new variables can be obtained by using Eq. (5) and substituting $C_q = \rho V_\infty^3 \dot{q}_w$ to yield

$$\dot{q}_w = -G'(0)/(P_r E_c) \tag{13}$$

where the minus sign indicates the direction of the heat flux.

Analytical Results

In this section exact solutions for the temperature profiles of both the fluid and particulate phases and the wall heat transfer coefficient will be reported. Since the flow is incompressible and the thermophysical properties of the dusty flow are assumed to be constant, the solution of the energy equations of both phases is uncoupled from the solution of the mass and momentum equations.

The solutions for the velocity profiles were reported earlier by Chamkha.[4] These were shown to be

$$F = 1 - \exp(-\lambda_1\eta), \quad F_p = 1 - \alpha/(\alpha + r_v\lambda_1)\exp(-\lambda_1\eta) \tag{14}$$

λ_1 being the absolute value of the negative root of the cubic equation

$$r_v\lambda^3 + (r_v^2 - \alpha)\lambda^2 - r_v[\alpha(1 + \kappa) + M^2]\lambda + M^2\alpha = 0 \tag{15}$$

Equations (8) and (9) can be combined into a third-order differential equation in terms of G. This can be shown to be

$$G''' + (r_v^2 P_r - \varepsilon)/r_v G'' - P_r\varepsilon(1 + \kappa\gamma)G' + E_c P_r[(F')^2]'$$
$$+ E_c P_r \kappa\alpha[(F_p - F)^2]' - E_c P_r \varepsilon/r_v (F')^2$$
$$- E_c P_r \kappa\alpha\varepsilon/r_v (F_p - F)^2 + M^2 E_c P_r[(F - 1)^2]'$$
$$- M^2 E_c P_r \varepsilon/r_v (F - 1)^2 = 0 \tag{16}$$

Upon substitution of Eq. (14), the solution of Eq. (16) subject to the last three terms in Eq. (11) can be shown to be

$$G = 1 + A\exp(-m\eta) + B\exp(-2\lambda_1\eta) \tag{17}$$

$$A = G_0 - 1 - B$$
$$B = E_c P_r \lambda_1^2/[r_v(\alpha + r_v\lambda_1)^2](2\lambda_1 r_v + \varepsilon)$$
$$\cdot[(\alpha + r_v\lambda_1)^2 - r_v^2\kappa\alpha] + M^2 E_c P_r/r_v(2\lambda_1 + \varepsilon)$$
$$m = [C_1 + (C_1^2 - 4D)^{1/2}]/2 > 0, \quad C_1 = (r_v^2 P_r - \varepsilon)/r_v$$
$$D = -P_r\varepsilon(1 + \kappa\gamma) \tag{18}$$

The solution for G_p can then be obtained from Eq. (9). This can be shown to be

$$G_p = 1 + \varepsilon A/(r_v m + \varepsilon)\exp(-m\eta) + \varepsilon B/(2\lambda_1 r_v + \varepsilon)$$
$$\cdot\exp(-2\lambda_1\eta) \tag{19}$$

The wall heat transfer coefficient is obtained by differentiating Eq. (17) once, evaluating the result at $\eta = 0$, and then substituting into Eq. (13). If this is done, one obtains

$$\dot{q}_w = (Am + 2\lambda_1 B)/(E_c P_r) \tag{20}$$

It should be pointed out that if M is formally equated to zero in Eqs. (17–20), the solutions reported by Chamkha[6] will be recovered.

Graphical Results

The closed-form solutions reported in the previous section were evaluated numerically for various parametric values. For brevity, few results will be presented graphically to illustrate the influence of the Hartmann number on the solutions.

Figure 1 shows representative fluid-phase temperature profiles for various values of the Hartmann number M. It can be seen from this figure that as M increases, the fluid-phase temperature G approaches the freestream temperature faster, causing the fluid-phase temperature gradient at the wall to increase. This increases the wall heat transfer coefficient $\dot{q}_w$, since it is directly proportional to the wall temperature gradient. This is evident from Fig. 2 which presents $\dot{q}_w$ for various values of the Hartmann number M and the temperature inverse Stokes number ε. The dotted lines appearing in Fig. 2 correspond to the thermal equilibrium limit between the two phases approached as $\varepsilon \to \infty$. Increases in the values of ε have the tendency to increase the interphase energy transfer between the two phases causing the wall heat transfer to increase. This is also apparent from Fig. 2. It should be mentioned that the particle-phase temperature G_p is also increased by an increase in the value of M.

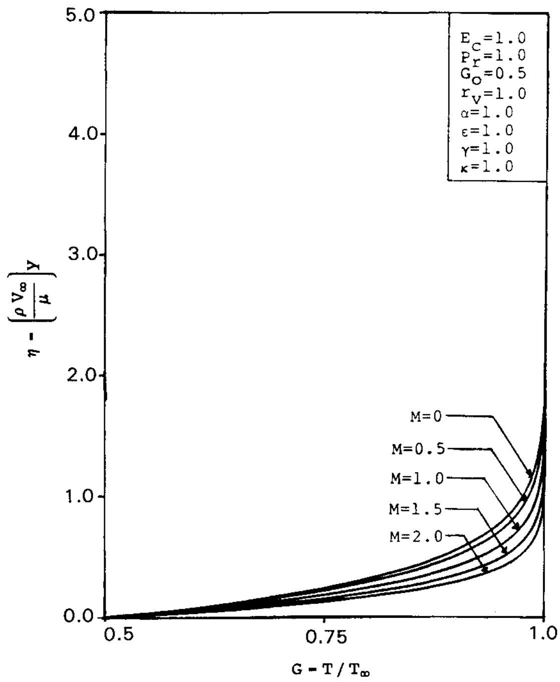

Fig. 1 Fluid-phase temperature profiles.

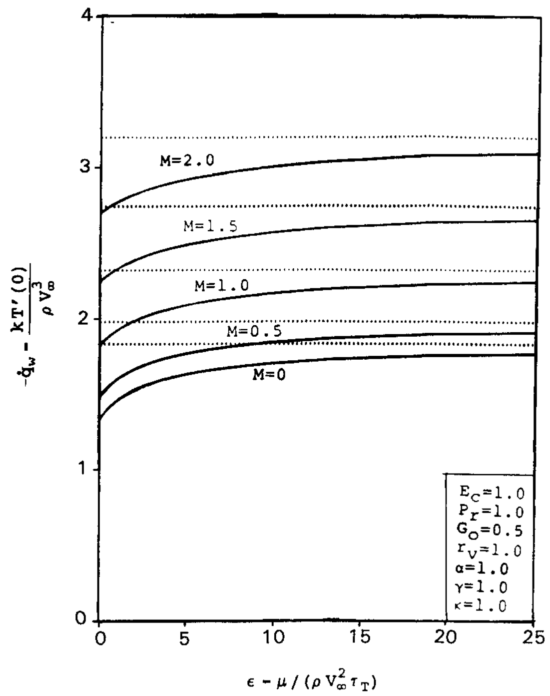

Fig. 2 Wall heat transfer vs ε.

Conclusion

The steady laminar flow solutions of particulates suspended in an electrically conducting fluid past an infinite porous flat plate presented in this article indicate the changes that will be brought about by a transverse magnetic field exhibiting relative motion with the plate. It was found that the heat transfer rate $\dot{q}_w$ increased as the strength of the applied magnetic field increased.

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Microsensors for High Heat Flux Measurements

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Introduction

MANY different methods have been developed for measuring surface heat flux in aerodynamic flows.[1-3] Unfortunately, heat flux gauges are often obtrusive and can cause surface disruptions which result in significant flow disturbances. Even if a smooth surface is maintained, however, the presence of a gauge still usually distorts the temperature field of the surface. This alters the heat flux that the gauge measures and it changes the thermal boundary layer, both of which cause measurement errors.

Figure 1 illustrates a typical heat flux gauge. The heat flux passes through the gauge into the surface and is proportional to the temperature difference $T_1 - T_2$ which is measured by temperature sensors. The temperature at the material surface, however, is different from the temperature at the surface of the gauge, causing the heat flux measured by the gauge to be in error. If an adhesive layer is used for attachment of the gauge to the surface, an additional temperature drop is created. For high-temperature flows the problem is compounded if the gauge is actively cooled. This usually creates a cold spot, which results in an artificially high measurement. As detailed by Neumann et al.[4] the reliability of the resulting heat transfer measurements is poor.

One way to eliminate the thermal disruption problem is to use passive thin films. When made sufficiently thin, the temperature drop across such a layer becomes negligible, compared to the overall temperature difference between the surface and the fluid. One method that has been used for many years is to infer the heat transfer from thin-film temperature measurements.[3,5] The heat flux is determined using an analytical model of the transient temperature response of the material. The method is best suited for short-duration flows (~milliseconds), although the test article can also be rapidly injected into a steady flow.[6]

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